Exploring Ratio by Looking at Prison Statistics
Lesson Packet #2 – March 2015
The Change Agent <changeagent.nelrc.org>

Introduction:
This lesson is meant to supplement instruction on ratio and is aimed at those who are comfortable or getting comfortable at Level C in the CCR standards. This lesson first explores ratio and then offers students the opportunity to think critically about ratio in the context of socially relevant data presented in two kinds of charts. Answer sheets are provided – for the teacher’s reference only or to share with students – your choice.

Objectives:
• to understand and be able to explain the difference between ratio and hard numbers
• to think critically about how ratio (and rate) and hard numbers are presented in charts

The Standards for Mathematical Practice:
• Make sense of problems and persevere in solving them (MP.1)
• Model with mathematics (MP.4)
• Use appropriate tools strategically (MP.5).

Level C Standards for Mathematical Content:
• Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (6.RP.1)
• Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)

Materials:
• This 2-page worksheet
• “A Skyrocketing Prison Population” by Alexis Greenblatt from The Change Agent Issue #40 “extras” at http://changeagent.nelrc.org/issues/issue-40/ (Answer sheet included.)
• “Incarceration Rates in OECD Countries” by Alexis Greenblatt from The Change Agent Issue #40 “extras” at http://changeagent.nelrc.org/issues/issue-40/ (Answer sheet included.)

Some Background for the Teacher (A Note about Rate and Ratio):
In math, rate and ratio are slightly different. Ratio compares two similar numbers -- such as the number of people in prison in relation to the total number of people in the population. A rate usually compares different units -- such as miles per hour. However, in common English usage, the terms are interchangeable. For more on rate and ratio, see http://mathforum.org or http://www.differencebetween.com/difference-between-rate-and-vs-ratio/

Steps:
1) Warm up: Let’s say you are trying to decide which GED class to take. You want to choose one with a very effective teacher. You ask your friends to find out what they can about the teachers, and here is what they report:
   Friend #1 says: “Ten students thought Teacher A was effective.”
   Friend #2 said: “Seven out of ten students thought Teacher B was effective.”
   Friend #3 said: “10% of the students thought Teacher C was effective.”

   Consider what information you get from each statement. Which would you guess is the most effective teacher? Why do you think that? Which class appears to have the least effective teacher? Which class is it hardest to make a judgment about? Why? What additional information would you need to make a better judgment?

2) Explain that ratio is a number given in relationship to another number. The Ratio 1 to 2 (1:2) can also be written as a fraction (1/2) or as a percent (50%).
3) Introduce the idea of “ratio” vs. “hard numbers.” Ask students to read the text in “The information given” column in the chart below and put an “R” for ratio or “HN” for hard number in the third column. Then ask students to read the math problem associated with the information given. If the problem can be solved with the information given, write Yes in the 5th column. If it can’t be solved, write No. If the problem can’t be solved, identify what information is missing and what you would need to know to be able to solve the problem.

<table>
<thead>
<tr>
<th>The information given</th>
<th>R or HN?</th>
<th>The math problem</th>
<th>Can you solve it with the information given? Y/N</th>
<th>What information is missing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1 out of 5 students brings an apple to school.</td>
<td>R</td>
<td>How many apples did the children bring to school?</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>2. I have 20 apples.</td>
<td>HN</td>
<td>I have twice as many apples as my friend.</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>3. Of every 3 people in our GED class, 2 pass the GED test on the first try.</td>
<td>R</td>
<td>How many people pass the GED in this class?</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>4. Six people in our class passed the GED test on the first try.</td>
<td>R</td>
<td>What percentage of people passed the GED test?</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>5. In 2006, over 7.2 million people in the U.S. were in prison, on probation, or on parole.</td>
<td>HN</td>
<td>What percent of the population was in prison, on probation, or on parole in the U.S. at that time?</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>6. In 2009, almost 5% of all black males were incarcerated.</td>
<td>HN</td>
<td>How many black males were incarcerated?</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>7. In 1953, there were about 100 people on death row. In 2001, there were about 3600 people on death row.</td>
<td>HN</td>
<td>From 1953 to 2001, the number of people on death row increased by how many times?</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>

Sources: United States Bureau of Justice Statistics (December 2009), [http://www.prisonpolicy.org/graphs/deathrow.html](http://www.prisonpolicy.org/graphs/deathrow.html)

Steps (continued):

4) After completing the above chart, students should explain ratio and hard numbers in their own words.
5) Now look at the two charts and answer the questions.
6) To conclude, students should reflect on why it is important to understand the difference between rate and hard numbers when they are being presented with statistics.

Find more Change Agent lesson packets at <changeagent.nelrc.org>, as well as PDFs of magazines on a wide variety of themes, articles in audio, CCR-aligned activities, and more. All of that is available for the price of an online subscription: $20 per teacher per year.
A Skyrocketing Prison Population

Alexis Greenblatt

First, share what you see in the graph. Then, identify specific things that you notice and/or questions you have. Finally, use the graph to answer the following questions.

1. From 1925-72, the number of incarcerated citizens was more or less at or below what number?

2. For about how many years did the number of incarcerated people in the U.S. remain relatively consistent?

3. From 1972-1988, the number of incarcerated people increased by approximately
   A. 2 times    B. 3 times    C. 4 times

4. Write a true sentence about what happened to the number of incarcerated people between 1980 and 2008.

5. In one sentence, summarize the data you see in this chart.

6. What might explain the dramatic shift in the number of people incarcerated?

7. Can you tell from this chart whether the ratio of incarcerated people to the whole U.S. population has also increased? What information would you need to answer that question?

**Source:** Bureau of Justice Statistics Prisoners Series. See <www.sentencingproject.org> for more statistics, information, and news.

Alexis Greenblatt teaches adult basic education at Springfield Technical Community College.

**NOTE:** a different version of this piece appeared in the print version of Issue #40 of The Change Agent.
Answers to questions about “A Skyrocketing Prison Population”:

First, share what you see in the graph.  
*Students may notice things like:*
- there are numbers of people on the y axis and years on the x axis
- that the graph is level for many decades and then rises very steeply.

Then, identify specific things that you notice and/or questions you have.  
*Students might notice and/or ask things like:*
- the numbers are in increments of 200,000
- the title refers to state and federal prisons. Does that mean it does not include other places that people are incarcerated, such as jails and juvenile detention centers?

Finally, use the graph to answer the following questions.

1. From 1925-72, the number of incarcerated citizens was more or less at or below what number?  
   Answer: 200,000

2. For about how many years did the number of incarcerated people in the U.S. remain relatively consistent?  
   Answers may vary. Possible answers include:  
   - For almost 50 years (between 1925 and 1975), the number of incarcerated people was between 100,000 and 200,000.  
   - For 32 years (from 1940 to 1972), the number of incarcerated people was fairly consistent, ranging between approximately 180,000 and 200,000.

3. From 1972-1988, the number of incarcerated people increased by approximately  
   A. 2 times  
   B. 3 times  
   C. 4 times  
   Answer: B. 3 times

4. Write a true sentence about what happened to the number of incarcerated people between 1980 and 2008.  
   Answers may vary. Possible answers include:  
   - From 1980-2008, the number of incarcerated people quadrupled.  
   - The prison population doubled from 1972 to 1982. Then it doubled again from 1982 to 1990 to 2008.

5. In one sentence, summarize the data you see in this chart.  
   Answers may vary. Here is one possible answer:  
   - The U.S. prison population was relatively stable for many years, but then it started to rise very rapidly starting in the 1980s.

6. What might explain the dramatic shift in the number of people incarcerated?  
   Students should have a chance to hypothesize. For example, they might ask if an increased criminal activity could partially explain the increase in the number of incarcerated people. They might guess that laws changed, causing more people to be imprisoned. Etc.  
   [Note that articles from Issue #40 of The Change Agent address trends like people waiting a long time for trial, indefinite detention for immigrants and others (pp. 8-9, 29), non-violent repeat offenders being jailed for life (p. 14), the “War on Drugs” (p. 16), stop and frisk (p. 18), prison labor (p. 20), and the “business” of mass incarceration (p. 24).]

7. Can you tell from this chart whether the ratio of incarcerated people in the U.S. has also increased?  
   Answer: No. You would need to know the population of the U.S. during various years to know if the ratio of incarcerated people went up or down during those years. For example, in 1925, the U.S. population was 115,829,000. Solving for x below, you find that the ratio of the incarcerated population to the whole population in 1925 was 86 per 100,000 or 86:100,000.

   \[
   \frac{100,000}{115,829,000} = \frac{x}{100,000}
   \]

   In 2008, The U.S. population was 304.1 million. Solving for x below, you find that the ratio of incarcerated people to the whole population in 2008 was 526 per 100,000 or 526:100,000.

   \[
   \frac{1,600,000}{304,100,000} = \frac{x}{100,000}
   \]

   In fact, the ratio of incarcerated people did increase between 1925 and 2008, but you can only figure that out by looking up what the total population was during various years.
Incarceration Rates in OECD Countries

Alexis Greenblatt

1) Let your eye roam around the chart. What do you notice?

2) Get more specific. What type of countries are listed? What countries are left out? Discuss what the Organisation for Economic Co-operation and Development is.

3) What does “rate per 100,000” mean? Discuss the difference between “rate” and “hard” numbers. Can you tell from this chart which countries have the most actual prisoners? (For more on “rate” vs. “ratio,” see the next page.)

4) Use the chart above to answer the following math questions:
   a) Round each country’s incarceration rate to the nearest 10, then answer the following questions.
   b) Which country incarcerates people at approximately twice the rate as the Netherlands does?
   c) Which country incarcerates people at half the rate as Turkey?
   d) The U.S. incarceration rate is about how many times greater than the country with the next largest incarceration rate?
   e) Which country has an incarceration rate that is 3 times greater than Norway’s?
   f) Poland has a higher incarceration rate than Mexico, but it may be that Mexico has more prisoners than Poland. Explain in your own words how this could be true.
   g) What information would you need to find out whether Mexico has more prisoners than Poland? Go find that information and answer the question: Which country has more prisoners—Mexico or Poland?

ANSWERS ON REVERSE.

Answers to questions about “Incarceration Rates* in OECD Countries”:

1) Students may notice that there are countries listed on the y axis and numbers listed on the x axis, that the majority of the countries are European, that there are no African countries on the list, that one line is much longer than all the other lines, etc.

2) The OECD says that it is a forum of countries that are committed to democracy and the market economy. See <www.oecd.org> for more information.

3) Rate per 100,000 means that for every 100,000 people in the population, there are x number of prisoners. “Rate” tells you the proportion of prisoners to total population. (You could also call this ratio*.) “Hard” numbers tell you the actual number of prisoners. To practice using rates, look up the populations of select countries and calculate the actual number of prisoners in that country.

4a) 40, 60, 70, 70, 70, 90, 90, 90, 90, 90, etc.

b) New Zealand

c) Switzerland

d) more than 3x greater than Turkey

e) Mexico

f) Mexico may have a larger population than Poland, so Mexico could have a lower incarceration rate but a larger number of actual prisoners.

g) To find out if this is true, you would need to know the populations of Mexico and Poland. After looking up the population of Poland (which is 38.5 million), you could set up an equation like this to find number of prisoners in Poland:

\[
\frac{x}{38,500,000} = \frac{224}{100,000}
\]

Cross multiply 224 by 38,500,000 and divide the result by 100,000 to find x. Answer: x = 86,240. This is the actual number of prisoners in Poland.

The population of Mexico is 122.3 million. Set up a similar equation to find the total number of prisoners:

\[
\frac{x}{122,300,000} = \frac{209}{100,000}
\]

Cross multiply 209 by 122,300,000 and divide by 100,000, and you will find that x = 255,607. This is the actual number of prisoners in Mexico. So, although Mexico has a lower incarceration rate than Poland, it has a higher number of actual prisoners.

A NOTE ABOUT RATE AND RATIO: In math, rate and ratio are slightly different. Ratio compares two similar numbers – such as the number of people in prison in relation to the total number of people in the population. It can be expressed as a fraction or a percent. A rate compares different units – such as miles per hour. However, in spoken and written English, the terms are sometimes used interchangeably. For example, this chart discusses incarceration “rates,” even though the “units” being compared are both people and so might more accurately be called a ratio. For more on rate and ratio, see “Dr. Math” at <http://mathforum.org/library/drmath/view/58042.html>.